Power Optimization For a Wind Turbine

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Renewable energy, such as Wind Turbines, plays a big role today in the world's power production. A recent report from the Intergovernmental Panel on Climate Change (IPCC) estimated that renewable energy production must ramp up from 20% of the global energy in 2018 to 67% by 2050 to stop the overall temperature from rising 1.5C across the world [1]. This increase in temperature can increase the problems that the world is facing today. The civilizations near the equator will find this very troubling as this will cause extremely high temperatures during the summers. The land will become more dry making agriculture very troubling as the crops' risk of dying increases. The same can be the same for an open body of water, increasing the chances of the body of water drying up. The demand for such resources will increase and the cost from the suppliers will continue to increase with the decreased amount they usually provide to the civilization. These extreme temperatures can lead to mass immigration which can put humans on a journey to another civilization taking them through areas that are not familiar regarding resources. The journey can lead to fatalities due to the lack of resources which we believe is the most troubling effect of climate change, putting humans at risk of immigration and leaving their lands. Any progress on innovations in renewable energy or improvements in current renewable power generation technology is needed. In fact, we believe there must be rapid improvements in this sector in order to meet the goal stated by IPCC for renewable energy production.

Climate Change | Multi-Objective Optimization | Power Generation | Renewable Energy | Wake Region | Wind Energy | Wind Turbines

I. Introduction

The current state of emissions will result in a temperature rise of 1.5C by 2040 found [1] by The Intergovernmental Panel on Climate Change (IPCC). Moreover, there is still a surplus of coal-based electricity that needs to decrease ~40% to a total of >7% of global energy production to halt the global warming process. As a result, there must be a rapid increase in technological advancements of renewable energy for us to stand a chance in this battle with climate change. About 20% of energy generated is from renewable energy, this needs to increase to 67% by 2050 [1]. Most of the renewable energy comes from wind and solar, because of their decreasing cost of electricity [2,3]. Advancements in size of the rotor diameter (D) have increased the energy generated by such wind turbines. There exist wind turbines that are offshore with 200m in rotor diameter generating copious amounts of energy. Increasing the energy generated for a single unit can help decrease the production needed for such wind farms, however, looking at the economics of these giants can help us see that this cannot be the only advancement to focus on. Such turbines are placed multiple kilometers (Km) apart to minimize aerodynamics losses between the array of turbines. Placing them in such a configuration will increase the cost of transmission lines, maintenance cost, and labor cost for transportation for having to travel further using multiple machines.

As a result, there needs to be a focus on increasing the efficiency of wind turbines. When optimizing for a turbine the focus is to maximize the power coefficient, which is a ratio of the energy the wind turbine is taking from the wind to the energy that is left in the wake region. Tackling this problem left researchers with a multiobjective optimization function. We will be using two major theories that have been researched regarding the power optimization of a wind turbine. Blade Element Momentum Theory (BEM) theory is a combination of the two fundamental theories that formulate the wind turbine: Blade Element Theory and Momentum Theory. The two fundamental theories were developed in the late 1860s - 1870s by William Froude and William J.M. Rankine respectively. In Blade Element Theory developed in 1878, you can evaluate the forces acting on the turbine by cutting the blade into elements and observing the force acting on the element which is a function of the design of the blade and the flow of the fluid through the turbine [4]. Momentum Theory developed in 1865 a global theory that adopts a macroscopic point of view, unlike Blade Element Theory which is a local view, to model the behavior of a column of fluid passing through a turbine [5].

It was not until 1926 that Hermann Glauert combined the two fundamental theories resulting in Blade Element

Momentum Theory which is based on two decompositions: (i) a radial decomposition of the blades and the fluid column, considered as concentric rings that do not interact with each other, and (ii) a decomposition of the fluid/turbine system into a macroscopic part via Momentum Theory and a local planar part via Blade Element Theory [8]. Given the particular location and size of a turbine, we are left optimizing design parameters of the twist angle and chord length to the corresponding wind speeds of the location. Note that there is research currently on the topic of optimizing wind turbines locations respectively to each other in a wind farm, however, our aim is to optimize the power coefficient of a singular turbine. Researchers are left with a multiobjective optimization problem. The goal of this paper is to explore the idea of creating a multi-objective optimization problem that will maximize the power coefficient of the turbine.

II. Blade Element Momentum Theory

The Blade Element momentum was introduced by Hermann Glauert in 1926. The model is used to describe the interaction between a turbine and the fluid flowing through it which is a combination of Blade Element Theory and Momentum Theory. The Theory aims to establish algebraic relations that explain the interactions between the flow of the fluid and the rotating blade of the turbine.

The BEM theory has some limitations. The flow is supposed to be horizontal, constant in time, and incompressible [8]. The left and right neighborhoods of the turbine have the same flow velocity.

We have a fixed blade element and a constant rotation speed Ω.

$$\lambda = \frac{\Omega r}{U-\infty}[9]$$

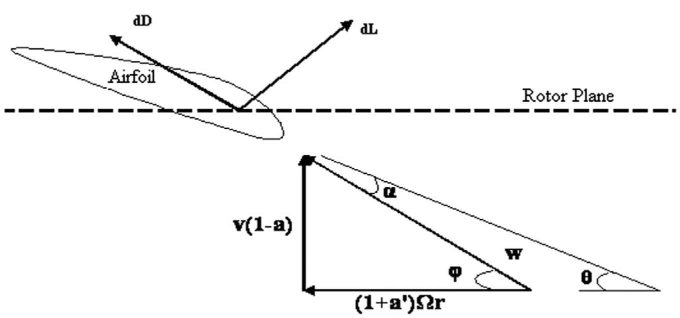

Fig 1: Forces acting on the blade.

The Figure above(Fig. 1) shows the forces acting on the blade. The variables mentioned in the figure are explained in the section below.

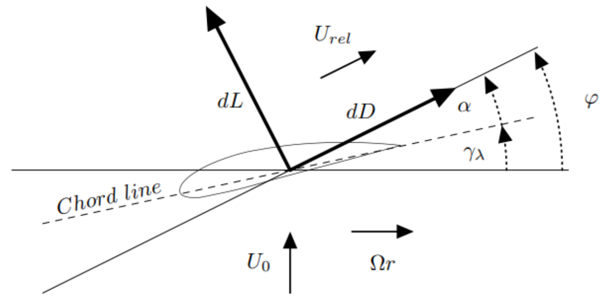

Fig 2: Blade element profile and corresponding variables.

As can be seen in the figures, the drag acts at an angle to the Rotor Plane, and we have lift acting perpendicular to the direction of the drag.

III. Problem Formulation

A. Variables

The BEM model consists of three primary variables a,a' and φ. These variables are correlated with a ring of fluid. The variable a refers to the axial induction factor and a' refers to the angular induction factor.

The angle φ refers to the relative angle deviation of the ring.

$$tan(\varphi) = \frac{1-a}{\lambda(1+a')}[8]$$

For a given blade profile we have Coefficients of Lift and Drag. The coefficients are denoted by C_L and C_D respectively. We have the chord for the blade c_λ.

We denote α, the parameter called the angle of attack as,

$$\alpha = \varphi - \gamma_\lambda[13]$$

Considering *dT* and *dQ* which is the infinitesimal thrust and torque. It is applied to blade element of thickness *dr*. The value for *dT* and *dQ* is given by[8],

$$dT = 4a\,(1-a)\,U^2_{-\infty}\,\rho\,\pi\,r\,dr$$

$$dQ = 4a'\,(1-a)\,\lambda\,U^2_{-\infty}\,\rho\,\pi\,r^2\,dr$$

The local thrust coefficient is given by,

$$C_T = 4a(1-a)$$

We try to achieve closed system equations and from our literature[8] we get,

$$\frac{a}{1-a} = \frac{\sigma_\lambda}{4sin^2\varphi}(C_L(\varphi-\gamma_\lambda)cos\varphi + C_D(\varphi-\gamma_\lambda)sin\varphi)$$

$$\frac{a}{1-a'} = \frac{\sigma_\lambda}{4\lambda sin^2\varphi}(C_L(\varphi-\gamma_\lambda)sin\varphi - C_D(\varphi-\gamma_\lambda)cos\varphi)$$

B. Simplified Model

The drag forces acting on the blade element are small enough that they do not contribute enough to the induced velocity physically. Therefore, we ignore the term C_D when we are calculating the induced velocities for the blade element[10].

The supposition above seems true for most of the cases. The main objective of the blade design is usually to minimize drag. We can safely ignore the coefficient of drag without the loss of generality, as we are looking to minimize the drag.

Considering $C_D = 0$ for the above-mentioned model we arrive at a simplified set of equations[8].

$$tan\varphi = \frac{1-a}{\lambda(1+a')}$$

$$\frac{a}{1-a} = \frac{1}{sin^2\varphi}\mu_L(\varphi)cos\varphi$$

$$\frac{a'}{1-a} = \frac{1}{\lambda sin\varphi}\mu_L(\varphi)$$

where,

$$\mu_L(\varphi) = \frac{\sigma_\lambda}{4}C_L(\varphi - \gamma_\lambda)$$

The above-mentioned equation is limited to the values of the axial induction factor which are less than 0.4. When the value of ‘a’ increases from 0.4 there is a generation for turbulent wake regions[11].

To account for this phenomenon we use the corrected model.

C. Corrected Model

When the value of ‘a’ increases to more than 0.4, a turbulent wake region appears in the flow[11]. We, therefore, now consider C_D as strictly positive and assume that there is a slow increase in this variable.

When designing for the blades of the wind turbine we consider the force from the blades acting on the flow as constant. But, in realistic scenarios, this is not true. Therefore, we use the Prandtl Tip function F_λ[12]. The function is given by,

$$F_\lambda(\varphi) = \frac{2}{\pi}cos^{-1}(exp(-\frac{B/2(1-r/R)}{(r/R)sin\varphi}))$$

We use the Prandtl Tip Function for the tip loss correction and get the following equations[8].

$$tan\varphi = \frac{1-a}{\lambda(1+a')}$$

$$\frac{a}{1-a} = \frac{1}{sin^2\varphi}(\mu^c_L(\varphi)cos\varphi + \mu^c_D((\varphi)sin\varphi) - \frac{\Psi((a-a_c)_+)}{(1-a)^2}$$

$$\frac{a'}{1-a} = \frac{1}{\lambda sin^2\varphi}(\mu^c_L(\varphi)sin\varphi - \mu^c_D((\varphi)cos\varphi)$$

where we have dimensionless functions,

$$\mu^c_L = \frac{\sigma_\lambda}{4F_\lambda(\varphi)}C_L(\varphi - \gamma_\lambda)$$

$$\mu^c_D = \frac{\sigma_\lambda}{4F_\lambda(\varphi)}C_D(\varphi - \gamma_\lambda)$$

We try to get another simplified version for this corrected model and reformulate the problem.

D. Optimization Model

Using the BEM model we try to maximize the power generated by the wind turbine.

The Shaft power output is given by[13],

$$P_s = C_p(1/2)\rho\pi R^2U_o^3$$

Since we have factors that are natural, we try to maximize C_P, which is the power coefficient.

C_P is given by,[8]

$$C_p(\gamma_\lambda, c_\lambda, \varphi, a, a') = \frac{8}{\lambda^2_{max}}\int_{\lambda_{min}}^{\lambda_{max}}\lambda^3J_\lambda(\gamma_\lambda, c_\lambda, \varphi, a, a')d\lambda$$

where,

$$J_\lambda(\gamma_\lambda, c_\lambda, \varphi, a, a') = F_\lambda(\varphi)a'(1-a)(1 - \frac{C_D(\varphi-\gamma_\lambda)}{C_L(\varphi-\gamma_\lambda)}tan^{-1}\varphi)$$

We consider λ independently on a discretization grid, the Prandtl tip function as 1 for simplicity, and we write a, a’ and J_λ in terms of φ. we get,[8]

$$a = 1 - \frac{sin\varphi.cos(\theta_\lambda-\varphi)}{sin\theta_\lambda}$$

$$a' = \frac{sin\varphi.sin(\theta_\lambda-\varphi)}{cos\theta_\lambda}$$

$$J_\lambda = \frac{sin^2\varphi.sin(2(\theta_\lambda-\varphi))}{sin2\theta_\lambda}$$

where, $\varphi \in [0, \theta_\lambda]$ and the design parameters are

$$\gamma_\lambda^* = \varphi^* - \bar{\alpha}$$

$$c_\lambda^* = \frac{8\pi\mu_G(\varphi^*)}{BC_L(\bar{\alpha})}$$

To obtain $\theta_\lambda \in [0, \frac{pi}{2}]$ using the simplified model we can use the following equation.

$$\theta_\lambda = tan^{-1}(\frac{1}{\lambda})$$

The optimization problem, therefore, becomes as follows, using the software MatLab to solve the multiobjective function with constraints. We will be evaluating different cases of λ, to obtain the optimal twist angle γ_λ^* and chord length c_λ^*. The three objective functions are a, a', and J_λ respectively. Using the three objective functions they will be assigned weights to create Pareto points. Once the maximum has been obtained for each of the three objective functions the optimal φ^* will be used to obtain the twist angle γ_λ^* and chord length c_λ^*. We can then calculate the power coefficient to be plugged into the shaft power output equation.

Our objective functions with constraints can be seen below.

$$\max f(\varphi) = w_1.a + w_2.a' + w_3.J_\lambda$$

Subject to,

$$0 \leq \varphi \leq \theta_\lambda$$

$$0 \leq \theta_\lambda \leq \frac{\pi}{2}$$

The method used in the objective function to create Pareto optimal solutions is called aggregate Objective Function (AOF) optimization. This method contains weights ($w_1, w_2,$ *and* w_3) that reflect the importance of each design objective. To plot the Pareto points we used MatLab's fmincon function with the given objective functions and constraints inside of a for loop running 100 iterations to plot 100 points of the objective functions. Once all the iterations have been run in the for loop, we will then extract the maximum values for each objective function and calculate the power coefficient and shaft power output.

IV. Conclusion

First, let us examine the relationship between the lift coefficient (C_L) and the angle of attack (α). As well, show the relationship of Lambda (λ) and Theta Lambda (θ_λ).

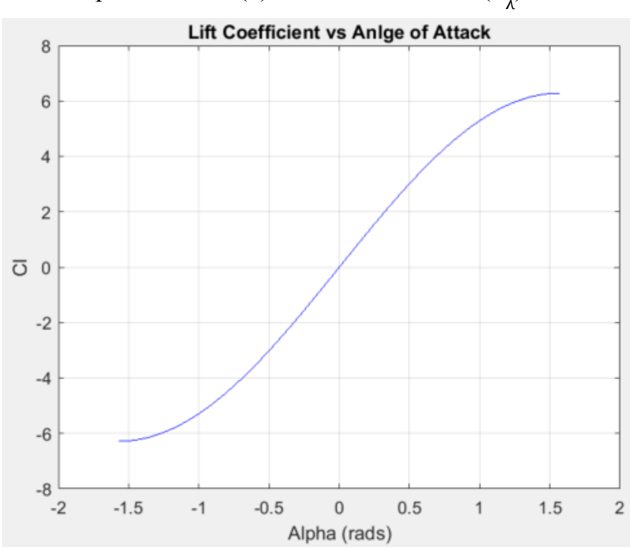

Fig 3: Lift Coefficient vs Angle of Attack

The above figure shows the relationship of the lift coefficient over an interval of the angle of attack to show how it changes the lift coefficient.

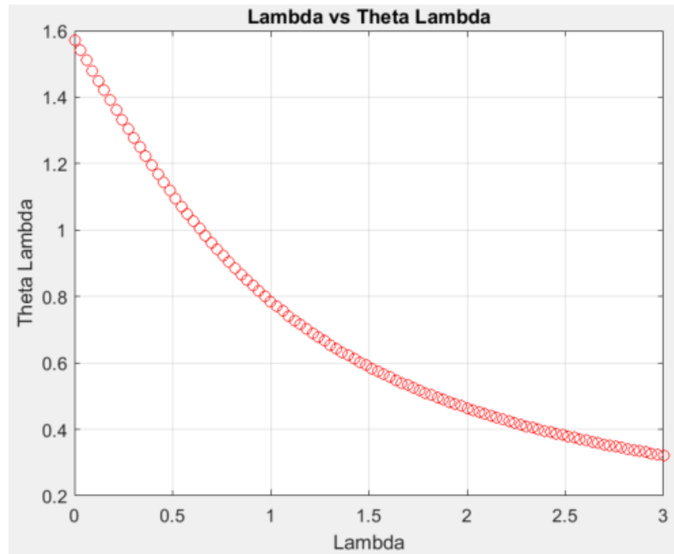

Fig 4: Lambda vs Theta Lambda

The above figure shows the relationship between the two variables over an interval lambda.

Referencing Fig 3, you can see the relationship in the a simplified model behaves as a sin function that is multiplied by a scalar. The lift coefficient is evaluated with alpha over an interval of $\alpha \in [0, \frac{pi}{2}]$. Referencing Fig 3, you can observe the relationship between the two variables is an inverse relationship which was evaluated over a period $\lambda \in [0, 3]$. Note that in our optimization code we have fixed values for λ and α. We will be optimizing the power coefficient for various cases of the two.

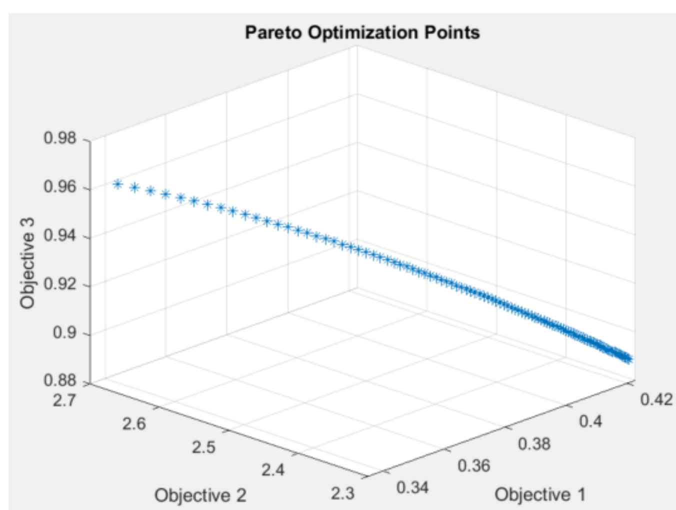

Fig 5: Pareto Optimization Points, λ = 0.25 and α = 5°

Using MatLab's Fmincon function to plot the points of the objective functions and show the relationship of the weights with the corresponding design objectives.

For λ = 0.25 and α = 5°

The optimal values for the objective function are: a = 0.335, a' = 2.66 , and J_λ= 0.9663. The optimal φ^* = 0.8528 rads which can be plugged into the γ_λ^*= 0.7386 rads and c_λ^* = 2.500 m. C_P = 0.1208 which is plugged into P_s= 2.231 KW

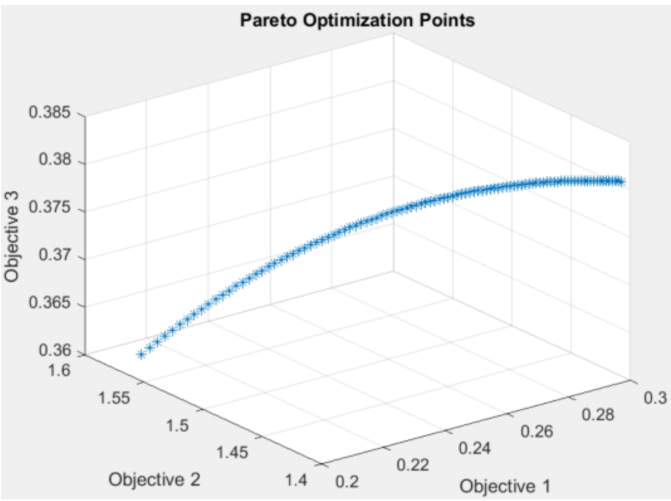

Fig 6: Pareto Optimization Points, λ = 0.5 and α = 5°
Using MatLab's Fmincon function to plot the points of the objective functions and show the relationship of the weights with the corresponding design objectives.

For λ = 0.5 and α = 5°

The optimal values for the objective function are: a = 0.2983, a' =1.403 , and J_λ= 0.3808. The optimal φ^* = 0.8258 rads which can be plugged into the γ_λ^*= 0.7386 rads and c_λ^* = 1.230 m. C_P = 0.1904 which is plugged into P_s= 3.517 KW.

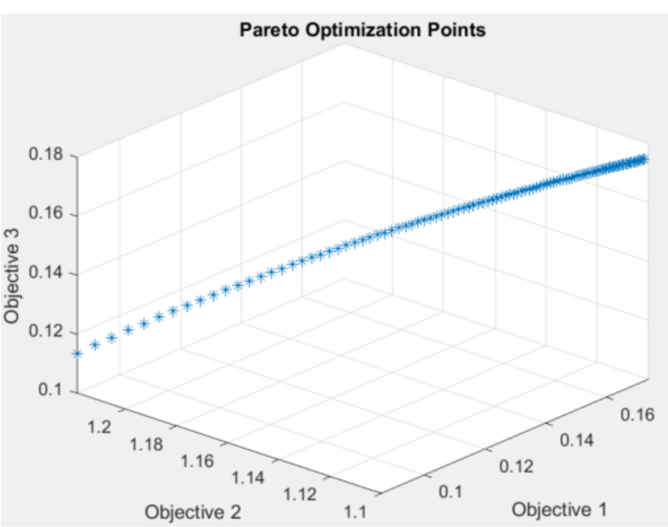

Fig 7: Pareto Optimization Points, λ = 0.75 and α = 5°
Using MatLab's Fmincon function to plot the points of the objective functions and show the relationship of the weights with the corresponding design objectives.

For λ = 0.75 and α = 5°

The optimal values for the objective function are: a = 0.1739, a' =1.101 , and J_λ= 0.1742. The optimal φ^* = 0.8258 which can be plugged into the γ_λ^*= 0.7386 rads and c_λ^* = 0.4580 m. C_P = 0.1960 which is plugged into P_s= 3.621 KW.

Observing the data collected off our three cases of different values of λ, we can see as λ approaches 1 leads to an increase in C_P which means the overall power generation of the turbine increases. In the case of the smaller value of λ = 0.25, we found that the optimal c_λ^* was the largest out of the other cases. The cost J_λ was higher due to the larger value of c_λ^*. The optimal φ^* was the same for all three cases due to the α staying constant throughout all the cases. We achieved our goal of optimizing for the power generation of a turbine by obtaining the optimal γ_λ^* and c_λ^* are their respective cases of various λ which are dependent on the location and weather patterns of the specified location.

V. Future Work

The Problem for Power Optimization is vast and there is so much research already being done. Our work for the topic is a droplet in an ocean and there is so much room for improvement.

Efficiency of a wind turbine is seen to be usually about 50% and ideally, we can design up to 59% efficiency, which is known as the Betz Limit[14]. The more the efficiency the more is the power generation of the singular turbine. This improved efficiency can make a huge impact when implementing this innovation across all turbines in a wind farm.

Some of the main areas of innovation are: Longer and lighter rotor blades – with some reaching 100 meters long, as seen in the offshore wind turbines. Blades with curved tips that are designed to take maximum advantage of all wind speeds. Blades that are better able to withstand the stresses of high-altitude wind and taller towers. Blades that can withstand higher stresses are placed in locations where overall the wind speeds are higher than average.

Yawing the turbines from facing the wind directly in a wind farm is another area of study that the main goal is to minimize the effect of the wake regions of the other turbines to the only directly behind it in a column to maximize the overall power generation of the wind farm by sacrificing the power generation of the leading turbine in their respective column. The goal is to find the optimal angle to yaw each turbine to maximize the overall power generation.

Wind turbines are designed to maximize the rotor blade radius to maximize power output. Larger blades allow the turbine to capture more of the kinetic energy of the wind by moving more air through the rotors. However, larger blades require more space and higher wind speeds to operate. As a general rule, turbines are spaced out at four times the rotor diameter. This distance is necessary to avoid interference between turbines, which decreases the power output. There is current research on the idea of optimally placing turbines in a given area based on a given terrain that are looking into different orientations to maximize power generation of the wind farm.

Those are just a few current research fields for renewable power generation. Future work for our research is to formulate a more complex objective function with more design variables and additional constraints for more rigorous computation. Another continuation is to involve studies of turbulent wind flow in the wake region and under what design parameters can optimally pull the most energy from those weather conditions.

This paper has opened our eyes to wind energy and the vast potential there is for improvements. We believe with rapid improvements and innovations in renewable energy, that the world can reach the goal set by IPCC for renewable energy production.

VI. Acknowledgment

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VII. References

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